TRANSIENT HEAT TRANSFER FROM SHRINKING LOX-DROP

BY

Nenad N. Kondic, Professor
Mechanical Engineering Department
University of Puerto Rico
Mayaquez, PR 00709

In order to achieve prescribed experimental conditions in wind tunnels, the nitrogen-oxygen mixture is enriched by injection of liquid oxygen (LOX) upstream of methane burners.

The objective of the study is to determine the LOX drop evaporation rate, which is dominated by heat transfer from the air stream (mass transfer mechanism can be later coupled with the solution from the present work). Since the concentration of drops in the surrounding gas is quite high, the concept of "infinite medium" cannot be utilized. Drop evaporation, as part of system's mass balance, is an important source-term in the finite-difference 2/3-dimensional network along and across the flow duct. This network is expressed in terms of laboratory-system (Eulerian) coordinates, while individual drop behavior, including its thermal history, is analyzed in Lagrangian coordinates.

The drop-to-carrier gas coupling involves not only the above mentioned mass balance, but also momentum/force interactions. The relative velocity between the drop and the surrounding gas affects the heat transfer to the drop; thus, it represents a time-dependent boundary condition. Drop velocity is obtained from the drop ballistics analysis. However, the initial ballistic analysis pertains to a constant-radius drop, and will now be applied to a shrinking drop. After the first-run solution of drop-size history, an improved ballistic analysis (of the shrinking drop) will be done, and the new relative velocity will be introduced in the drop heat transfer cycle. The result will be a more accurate drop-size history. After a number of iterations, the joint/coupled ballistic and thermal analysis of the individual drop will be completed. The next step is the mass- and energy-coupling of a multitude of drops (as sources of O_2) and the surrounding gas in the Eulerian network. This task, also involving iterations, is the continuation of the present study. The final analysis will be directed at the gas-to-gas turbulent diffusion, and will include the whole-time history after LOX injection.

The individual drop thermal history includes two distinct phases:

I.   Heat transport with a constant-drop radius, but a variable (increasing) drop-surface temperature (internal drop temperature is also increasing). This phase ends at time, θ_c when the surface reaches t_c, the critical temperature of LOX.

II.   Heat transport (including conduction and convection) with a variable drop size, i.e., radius, $r_d = f(\theta)$, but with a steady drop-surface temperature, $t_s = t_c$. The internal temperature field depends on time and the relative radius, $\rho = r/r_d$. The drop shrinks during Phase II, because the spherical surface at t_c penetrates as an isothermal front into the drop, "shedding-off" the outside layers, with $t > t_c$. When r_d reaches zero, the drop has ended its existence as a discrete, discontinuous entity.

Although the above description relates to drops receiving heat from the environment, the analysis results allow for the reversal of the heat flux direction. Also, the solution applies readily (or with minor adaptations) to solid spheres, various pairs of components, at diverse P/T conditions (e.g., for fuel injection in internal combustion engines).

The solution approach was aimed at obtaining/delivering primarily analytical solutions, because they offer generality, simpler and more conclusive parametric and feasibility studies. Also, for the same accuracy, they usually require much shorter computing time.

A survey of cases is given below, showing a variety of real situations; only for few of them the solutions either exist in available literature or can be adapted from published solutions. Most solutions are derived during the performance of this work; in particular, this applies to variable drop-radius cases, as well as to variable thermal conductivity and diffusivity of drops, when these are expressed as products of functions (see notes 2 and 3.).

I.   Constant Drop Radius: Nu = Nusselt group ("Number") constant or time dependent; for either Nu, drop thermal conductivity, k can be constant or temperature dependent. For all Nu and k options, thermal diffusivity, α can be constant or temperature dependent.

II.   Time Dependent (Diminishing) Drop Radius: Same alternatives (cases) as above, except that Nu cannot be constant: by definition, it is proportional to a (variable) drop radius.

Notes: 1. Variable Nu means a time dependent boundary condition (through the heat transfer coefficient), and/or a variable drop radius.

2. By splitting the temperature dependency of $\alpha(t)$ into $\alpha_1(r, \text{ or } \rho) \cdot \alpha_2(\theta)$, separation of variables can be achieved, i.e., $t(r, \text{ or } \rho, \theta) = R(r, \text{ or } \rho) \cdot T(\theta)$.

3. Similarly, using $k(t) = k_1(r, \text{ or } \rho) \cdot k_2(\theta)$, the time-dependent boundary condition can be replaced by the steady one, with separated variables in the temperature field solution.

4. For most cases, the solution for the temporal temperature function T, is an explicit, finite form exponential function, while the radial component of the temperature function, R, is the solution of a linear second order differential equation, easily transferrable into a Riccatti-type first order differential equation, reference 1.

   Items 2 and 3 were derived in course of this task, as well as several analytical adaptations of existing (published) solutions found in references 2 and 3.

REFERENCES:

1. Kamke, E., "Differential Equations" (German Language Edition), Becker and Erler, Kom-Ges, Leipzig, 1943/44.

2. Carslaw, H. S. and Yaeger, J. C., "Conduction of Heat in Solids," Oxford at the Clarendon Press, Second Edition, 1967.

3. Schneider, P. J., "Conduction Heat Transfer," Addison-Wesley Publishing Co., Inc., Reading, Mass, Second Printing, September 1957.

Figure: History of Drop Size, Temperatures and Relative Velocity (Qualitative Representation)

$\rho = r/r_d$

Phase I: $r_d = r_b = $ const

Phase II: $r_d = f(\theta)$

$\theta = \theta_u - \theta_c$

$\theta_u = $ universal time

$\theta = $ time during Phase II.